NONLINEAR INTERACTIONS OF LANGMUIR WAVES IN A WEAKLY INHOMOGENEOUS PLASMA

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Kinetic equations for the scattering of the waves of the one-dimensional spectrum by plasma particles are obtained for a weakly inhomogeneous plasma. The equation for the evolution of the spectrum of the short waves $[k^2 > (m_e/m_i) D_e^{-2}]$ trapped in the inhomogeneities of the plasma density differs significantly from the kinetic equation for the waves in a homogeneous plasma. The problem of localization on the spectrum of the Langmuir waves in regions near the minima of the plasma density is also considered. A solution of the kinetic equation for the waves, which describes this process, is obtained.

A number of papers [1-3] have dealt with the influence of a weak inhomogeneity of plasma density on the effective interaction of particles and waves in a plasma. The presence of an inhomogeneity in real experiments may greatly distort the dynamics of such processes in comparison with the model conditions of the homogeneous plasma. Thus, in [1] it is indicated that an appreciable change takes place in the spectrum of the Langmuir waves generated by an electron beam as a result of the existence of a weak inhomogeneity in the plasma density in the beam propagation direction. The method of [1] can be applied to an analysis of the nonlinear interaction of Langmuir waves of a one-dimensional spectrum.

Following [1], we consider a one-dimensional inhomogeneity of the plasma density in the form

$$n(x) = n_0(x_0) + \Delta n(x) \tag{0.1}$$

where $n_0$ is the average value of the concentration and $\Delta n(x)$ is a small time-independent deviation on the concentration from the mean value. The spatial scale of the inhomogeneity $a$ is assumed to be much larger than the characteristic wavelengths of the spectrum under consideration,

$$a \gg \lambda \tag{0.2}$$

Under this condition, the Langmuir waves in the plasma can be described as a superposition of quasiparticles (wave packets) whose distribution function satisfies the Liouville equation

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial k}\frac{\partial N}{\partial x} - \frac{\partial \omega}{\partial x}\frac{\partial N}{\partial k} = 2\gamma N \tag{0.3}$$

where $N = N(k, x, t)$ is the spectral density function of the quasiparticles, $\omega = \omega(k, x)$ is the solution of the dispersion equation, and $\gamma = \gamma(k, x, t)$ is the increment of the nonlinear scattering of the Langmuir waves by the plasma particles, for in the absence of intense nonpotential oscillations in an isothermal plasma, the conservation laws forbid three-plasmon processes for Langmuir waves of a one-dimensional spectrum (it is assumed that the linear dampening is exponentially small).

Near the minimum density, the dispersion equation for the Langmuir waves takes the following form:

$$\omega^2 = \omega_{pe}^2(x_0)\left(1 + \frac{\Delta n}{n_0}\frac{(x-x_0)^2}{a^2}\right) + 3k^2 v_{Te}^2 \tag{0.4}$$

$$v_{Te}^2 = \frac{T_e}{m_e}, \quad \omega_{pe}^2(x_0) = \frac{4\pi e^2}{m_e} n(x_0)$$

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Here $x_0$ is the coordinate of the minimum of the density, $\Delta n$ is the depth of the "inhomogeneity well" (henceforth referred to simply as "well"), and $a$ is its width.

When moving along the trajectory

$$\omega(k, x) = \omega(k_0, x_0) \tag{0.5}$$

quasiparticles with a sufficiently small wave number

$$k_0^2 < \frac{1}{3}\frac{\Delta n}{n_0}\frac{1}{D_e^2}, \quad D_e^2 = \frac{v_{Te}^2}{\omega_{pe}^2} \tag{0.6}$$

(the subscript 0 will henceforth pertain to quantities specified at the center of the well) experience reflection from the walls of the well at points where

$$k(x, k_0) = 0 \tag{0.7}$$

[the function $k(x, k_0)$ is obtained from the trajectory (0.5)].

We shall henceforth call such quasiparticles trapped, and their trajectories spaces are finite. The influence of the inhomogeneity on the process of nonlinear scattering will be appreciable if

$$a \ll \frac{1}{\gamma}\left|\frac{\partial\omega}{\partial k}\right| \tag{0.8}$$

and in the opposite case the process will terminate before the inhomogeneity has time to greatly distort the trajectory of the quasiparticle in the (k, x) phase space.

According to [4], the maximal nonlinear increment is determined by scattering by ions, and its order of magnitude is

$$\gamma_m \approx \omega_{pe} W / nT_e, \quad W = \omega_{pe}\int N dk \tag{0.9}$$

where W is the energy density of the Langmuir waves. Taking (0.8) and (0.9) into account, we can write down the inequality

$$W / nT_e \ll D_e / a \tag{0.10}$$

which imposes a limitation on the energy density of the Langmuir oscillations, the interaction of which with the particles is greatly influenced by the inhomogeneity.

The condition for the applicability of the nonlinear equations

$$N_{De}^{-1} \ll W / nT \ll 1, \quad N_{De} = {}^4/_3\pi n D_e^3 \tag{0.11}$$

in conjunction with (0.10), imposes on the plasma density, for which the analysis that follows is valid, the upper bound

$$n \ll 3\cdot 10^{-2} a^{-1} (T_e / e^2)^2 \tag{0.12}$$

## 1. Averaged Kinetic Equation for Waves

According to [1, 2], the kinetic equation for the waves, averaged over the inhomogeneities, takes the form

$$\partial N/\partial t = 2\langle\gamma\rangle N \tag{1.1}$$

For trapped quasiparticles, this equation describes the change in the number of plasmons at any point of the trajectory of their motion (e.g., at the minimum density), and the spectra with respect to k and $\omega$ are rigorously related at this point by Eq. (0.5)

$$\langle\gamma\rangle = \int_{x_1}^{x_2} \gamma\left(\frac{\partial\omega}{\partial k}\right)^{-1} dx \Big/ \int_{x_1}^{x_2}\left(\frac{\partial\omega}{\partial k}\right)^{-1} dx \tag{1.2}$$

Here $x_1$ and $x_2$ are the turning points of the plasmon (Fig. 1), and are determined from (0.7), while $\partial\omega(x)/\partial k$ is determined from (0.5).

In addition to [1, 2], it is necessary to make the following remarks with respect to the nonlinear scattering: since

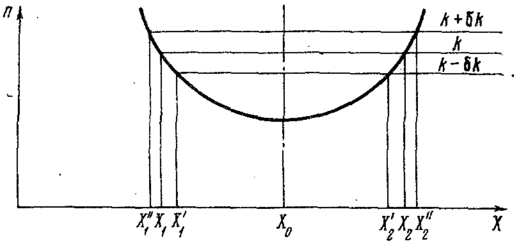

Fig. 1

$$\gamma(\mathbf{k}) = \int w(\mathbf{k}, \mathbf{k}') N(\mathbf{k}') d\mathbf{k}' \tag{1.3}$$

where $w(\mathbf{k}, \mathbf{k}')$ is the scattering matrix element, and since the region in the well where quasiparticles with $|\mathbf{k}_0'| < |\mathbf{k}_0|$ can exist is smaller (Fig. 1) than the region where quasiparticles with wave number $k_0$ exist, the integrals in (1.2) must in fact be taken between the closest limits of the range in which the two interacting waves exist.

Owing to the change of the phase volume in k space when moving the trajectory (0.5), it is convenient to change over in (1.3) to integration with respect to $\omega$ by using relation (0.5). Taking into account the inequality (0.8) and the symmetry of the well with respect to the point of the minimum density, we can write

$$\langle\gamma(\mathbf{k})\rangle = \int d\omega' N[\mathbf{k}'(\omega')]\left[\int_{x_0}^{x_2} w(\mathbf{k}, \mathbf{k}')\left(\frac{\partial\omega}{\partial\mathbf{k}}\frac{\partial\omega'}{\partial\mathbf{k}'}\right)^{-1} dx : \right.$$

$$\left. : \int_{x_0}^{x_2}\left(\frac{\partial\omega}{\partial k}\right)^{-1} dx\right] \equiv \int d\mathbf{k}' \langle w(\mathbf{k}, \mathbf{k}')\rangle N(\mathbf{k}') \tag{1.4}$$

The integration in (1.4) can be carried out by making the change of variables

$$y \equiv \frac{k}{k_0}, \quad dx = -\frac{y\,dy}{(1-y^2)^{1/2} b}, \quad b^2 = \frac{\Delta n}{3n_0}\frac{\omega_{pe}^2(x_0)}{k_0^2 v_{Te}^2} a^{-2}$$

in which case the points $x_0$ and $x_2$ go over into 1 and 0, respectively. If the integration limits are determined by the k' trajectory, then the upper limit will be $x'_2$, which is the point of reflection of the k' wave and corresponds in terms of the new variables to the point $[1-(k_0'/k_0)^2]^{1/2}$.

In scattering by particles we have, in accordance with [4],

$$\gamma_\alpha(\mathbf{k}) = B_\alpha \int d\mathbf{k}' N(\mathbf{k}') \frac{\omega'-\omega}{|\mathbf{k}'-\mathbf{k}|} \exp\left[-\frac{1}{2}\left(\frac{\omega'-\omega}{|\mathbf{k}'-\mathbf{k}| v_{T\alpha}}\right)^2\right]$$

$$v_{T\alpha}^2 = T_\alpha / m_\alpha \qquad (\alpha = i, e) \tag{1.5}$$

$$B_\alpha = (\pi)^{1/2} / 32 D_e^2 n m_e v_{T\alpha} \tag{1.6}$$

Recognizing that neither $\omega$ nor $\omega'$ varies on the trajectory (0.5), and replacing the exponential by the Heaviside $\vartheta$-function

$$\exp(-x^2) \approx \vartheta(1-x^2), \qquad \vartheta(x) = \begin{cases} 1, & x \geqslant 0 \\ 0, & x < 0 \end{cases} \tag{1.7}$$

we can carry out the entire integration and obtain for $\langle\gamma(\mathbf{k})\rangle$ the following expression:

$$\langle\gamma_\alpha(\mathbf{k})\rangle = B_\alpha \int d\mathbf{k}' k' N(\mathbf{k}') \frac{6v_{Te}^2}{\omega+\omega'} \vartheta\left[1 - \frac{1}{2}\left(\frac{\omega'-\omega}{|\mathbf{k}'-\mathbf{k}| v_{T\alpha}}\right)^2\right] \times$$

$$\times \frac{1}{\pi}\begin{cases} -\{\pi - \arcsin[1-(k_0'/k_0)^2]^{1/2}\}, & (\mathbf{nn}') = 1 \\ -\arcsin[1-(k_0'/k_0)^2]^{1/2}, & (\mathbf{nn}') = -1 \end{cases} |\mathbf{k}_0'| < |\mathbf{k}_0|$$

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$$\mathbf{n} = \mathbf{k}/|\mathbf{k}|, \qquad \mathbf{n}' = \mathbf{k}'/|\mathbf{k}'| \tag{1.8}$$

We note that none of the parameters characterizing the shape of the density-inhomogeneity well is contained in the final result.

The change of the average matrix element $\langle w(\mathbf{k}, \mathbf{k}')\rangle$ (1.4) in comparison with $w(\mathbf{k}, \mathbf{k}')$ in a homogeneous plasma is trivial when $|\mathbf{k}'| \ll |\mathbf{k}|$.

In this case

$$\langle w(\mathbf{k}, \mathbf{k}')\rangle = k' k^{-1} w(\mathbf{k}, \mathbf{k}')$$

since, unlike the homogeneous plasma, where the k wave interacts on the entire length of its existence with the k' wave, the interaction in a weakly inhomogeneous plasma takes place only on the path of the k' wave, which is smaller by a factor k/k' (according to (0.4), the ranges in the well are related like the wave numbers at the minimum-density point).

The averaged kinetic equation (1.1), with an increment of the types (1.8), describes the change of the number of quasiparticles at the level [on the trajectory (0.5)]. Unlike the case of a homogeneous plasma, where the description pertains to the spectral density per unit volume, the normalization ratio is also suitably altered in this case.

Since

$$L(k) \equiv \alpha k \qquad \left(\alpha = \left(\frac{3n_0}{\Delta n} aD_e\right)^{1/2}\right)$$

where L(k) is the length of the quasiparticle trajectory in the well, it follows that the number of waves on the trajectory is

$$P(\mathbf{k}) = L(k) N(\mathbf{k})$$

and Eq. (1.1) can be written in the form

$$\frac{\partial P(\mathbf{k})}{\partial t} = P(\mathbf{k}) \int d\mathbf{k}' P(\mathbf{k}') \varphi(\mathbf{k}, \mathbf{k}'), \quad \varphi(\mathbf{k}, \mathbf{k}') = -\varphi(\mathbf{k}', \mathbf{k})$$

Here $\varphi$ (k, k') depends already on the shape of the well, and its form can be easily obtained from a comparison of (1.1) and (1.8).

## 2. Nonlinear Scattering by Ions

According to [4], in the spectral region

$$k > (m_e/m_i)^{1/2} D_e^{-1} \equiv k^* \tag{2.1}$$

the predominant effect in the direction of Langmuir waves with ions is scattering through an angle $\pi$ with a small change of the modulus of the wave vector. In this case, owing to the difference, albeit small, between the ranges in the well, the kinetic equation for the wave, which takes into account the predominant interaction of waves having close wave numbers, acquires an additional term.

Let

$$\mathbf{k}' = -\mathbf{k} + \delta\mathbf{k}, \quad (\mathbf{k} \cdot \delta\mathbf{k}) / |\mathbf{k}| |\delta\mathbf{k}| = 1$$

Then

$$\frac{\partial N(\mathbf{k})}{\partial t} = \beta_i N(\mathbf{k}) \int d\mathbf{k}' N(\mathbf{k}') (\mathbf{k}' - \mathbf{k}) = -\beta_i N(\mathbf{k}) \int_0^{k^*} d(\delta k) \times [N(-\mathbf{k} + \delta\mathbf{k}) - N(-\mathbf{k} - \delta\mathbf{k})] \delta k \tag{2.2}$$

but

$$N(-\mathbf{k} + \delta\mathbf{k}) - N(-\mathbf{k} - \delta\mathbf{k}) = N^+(-\mathbf{k}) - N^-(-\mathbf{k}) + 2 \frac{\partial N(k)}{\partial \mathbf{k}}\bigg|_{-\mathbf{k}} \delta\mathbf{k} \tag{2.3}$$

where

$$N^+(-\mathbf{k}) = N(-\mathbf{k} + \delta\mathbf{k}) - \frac{\partial N}{\partial \mathbf{k}}\bigg|_{-\mathbf{k}} \delta\mathbf{k}, \quad N^-(-\mathbf{k}) = N(-\mathbf{k} - \delta\mathbf{k}) + \frac{\partial N}{\partial \mathbf{k}}\bigg|_{-\mathbf{k}} \delta\mathbf{k}$$

In a homogeneous plasma Eq. (2.2) takes the form

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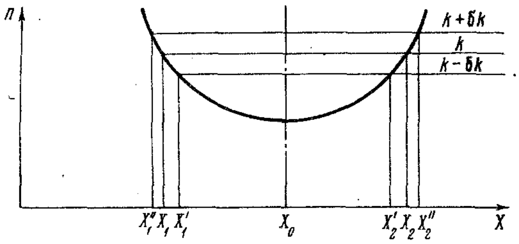
In the case of an inhomogeneous plasma the expansion (2.3) does not reduce merely to a differential term, since the spatial regions of the existence of the quasiparticles, which are characterized by the numbers $N^+(-\mathbf{k})$ and $\bar{N}(-\mathbf{k})$ of the waves, differ by an amount $\sim k^* a/k_0$, as the result of which the difference

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The scattering of waves of a spectrum localized in a well is described in terms of the number of waves on the trajectory. The need for modifying Eq. (2.4), which corresponds to a homogeneous plasma, becomes obvious if it is recognized that in an inhomogeneous plasma the number of waves per unit volume is not conserved in scattering processes.

In conclusion, the author thanks A. S. Kingsep for suggesting the problem and for directing the work.

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